Bubbles are rational

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Abstract

As we show using the notion of equilibrium in the theory of infinite sequential games, bubbles and escalations are rational for economic and environmental agents, who believe in an infinite world. This goes against a vision of a self regulating, wise and pacific economy in equilibrium. In other words, in this context, equilibrium is not a synonymous of stability. We attempt to draw from this statement methodological consequences and a new approach to economics. To the mindware of economic agents (a concept due to cognitive psychology) we propose to add coinduction to properly reason on infinite games. This way we refine the notion of rationality.

Keywords: economic game, infinite game, sequential game, bubble, escalation, microeconomics, speculative bubble, induction, coinduction.

> There is always some madness in love. But there is also always some reason in madness.
>
> *Friedrich Nietzsche*
> Thus Spoke Zarathustra (1898)

Traders speculate with no limit, countries bankrupt, world consumes energy like crazy. Are our models of a self-regulating economy, which rejects bubbles as unlikely, adapted? Are we sure to understand how agents act?

The 2008 subprime crisis has shed light on two problems. One, the crisis is not due to mad actors, but to the interaction of intelligent actors. Two, the current tools of economics, based on concepts elaborated in the middle of the previous century are out of date.

Here we focus on a well known phenomenon, namely escalation[1] whose rationality has been questioned and even refuted as paradoxical. Escalation consists in taking with no limit a sequence of decisions with heavier and heavier consequences. This headlong run strikes today the economy, the finance and the social and environmental development and is a characteristic of financial bubbles. This apparent irrationality has been illustrated by Newton after the emergence of one of the first financial crisis, namely the South Sea Bubble when he said that "he could calculate the motions of erratic bodies, but not the madness of a multitude". In escalation, agents behave absolutely rationally, provided they believe in the endless availability of natural or financial resources. Indeed the trader or the investor is rational because he reasons in his own world which he thinks infinite,

[1] A well known phenomenon in the literature from Macbeth to Madame Bovary

because he aims at maximizing his profits and because he believes that he can create money with no limit. Amazingly a person involved in an escalation can bid indefinitely with no respect for his loss. For an external observer, the decision taker implied in an escalation seems to have lost his common sense, but from the point of view of the decision taker himself locked in his closed world he is perfectly rational. The consistence of this attitude will be proved by a subtle and correct reasoning on infiniteness. This level dependent perception is probably what distinguishes instrumental rationality from epistemic rationality (see Section 9). Hence, an agent who captures both internal and external vision and takes both into account is *epistemicly rational* whereas an agent prisoner of the system is only *instrumentally rational.* The first agent will foresee the bubble outburst, when the second agent will remain blind.

At last, we do not say that an escalation followed by a collapse which can be its consequence is unavoidable, but we claim that it is plausible since escalation is supported by a rational behavior.

1 Toward new tools for analyzing systems

It is human nature to think wisely and to act absurdly
Anatole France
Le livre de mon ami
The book of my friend

In this article, we study systems with agents, where a system is an organization in which elementary entities able to reason are called "agents". In what follows, we use also the term "player", by analogy with games which we will use as our paradigm. According to its concern, an agent takes decisions which are consequences of her preference or of her appreciation of what she can gain. Of course, those choices influence the global behavior of the system. A behavior, we are interested in, is an *equilibrium*, in which the decision of the agents are made in order to maximise their returns. In that sense, equilibrium is synonymous of *stability.* But as we will show, this can coincide with a quick evolution of the main parameters of the system,[2] as for instance the quick raising or decreasing of a price. Thus an equilibrium can yield a big instability of the main parameters of the system, as this is the case in escalation, a concept we will focus on. Roughly speaking we investigate sequences *equilibrium-decision, equilibrium-decision*, etc. and instability can result as an outcome of these sequences. In infinite games, equilibrium is no more synonymous of stability. This propensity of the agents toward optimization is what people call "rationality", in other words agents are gifted with a reason which they use for their advantage in their decisions. But rationality has two faces, according to how it is viewed. Indeed it can be viewed from inside the system or from outside, that is from a local or from a global perspective. These two points of view lead to two opposite statements. Escalation which is specifically irrational from a holistic point of view is rational from a reductionist one. This phenomenon raises many paradoxes. First a local rationality, this of the agents taken individually, can result in a total irrationality when the system is taken as a whole. Since the agent is squeezed in her world, it will be difficult for an external observer to convince her of her mistake. When a observer affirms the rationality or the irrationality of an agent he should tell at which level he stays. Second, a system founded on equilibria can be

[2] In general relativity, a black hole is the result of an equilibrium, whereas we know how extreme its behavior is.

chaotic and a outburst can be a consequence of a sequence of equilibria. Therefore, the expression "agents in equilibrium when taking decision" does not mean "a system with a slow and regular evolution".

Chaos and escalation are not well-understood in scientific disciplines (among others economics) grounding part of their mathematical model on multi-agents systems. Therefore, we feel very close to Jean-Philippe Bouchaud who claims that "Economics needs a scientific revolution" [3] or David Colander who announces the end of the economics as we know it and calls for a refoundation. New conceptual tools should be used, based on logic as developed by another science of large systems, namely computer science, on top of which coinduction and coalgebras lie.

2 A case study: McDonald's Restaurants vs Morris & Steel

Tell me what you eat, I tell you who you are.
Anthelme Brillat-Savarin,
La physiologie du goût (The physiology of Taste)

An interesting case of escalation is this which opposed McDonald's Corporation to two English environmental activists, David Morris and Helen Steel [4] in the infamous so-called *McLibel Trial*, also known as *McDonald's Restaurants v Morris & Steel.* Clearly McDonald's management has a well-thought-out policy, knows how to argue and takes advice from the best lawyers. The trial which latest ten years was the longest-running case in the English history. The facts started by a small leaflet campaign, making allegations, not all asserted. Helped by the stars of the bar association and firmly grounded on a supposed "credible threat" (see Section 8), namely that the powerful MacDonald's scares the activists and can support an endless case, the company suited the activists for libel. Two of them, David Morris et Helen Steel, decided to defend their case. At the end of a very long trial, the judge ruled mostly against the defendants, because some allegations of the pamphlet could not be proved. But in retrospect the company lost. Specifically, the judge ruled that McDonald's endangered the health of their workers and customers by "misleading advertising", that they "exploit children", that they were "culpably responsible" in the infliction of unnecessary cruelty to animals, and that they were "antipathetic" to unionization and paid their workers low wages. This was evidenced in court by a media circus. Eventually, the defendants took the case to the European Court of Human Rights who ruled against the British government because UK laws had failed to protect the public right to criticize corporations whose business practices affect people's lives and the environment; they also ruled that the trial was biased because of the defendants' comparative lack of resources and what they believed were complex and oppressive UK libel laws. Eventually the law was changed. It has been estimated that McDonald's Company has lost 10 000 000 £ with further consequence, when the activists spend 30 000 £, supported by subscription and the compensation ruled by the European Court of Human Rights. This is a typical case of escalation. Although the company saw that it was trapped and lost in image, it could not be defeated in the court and kept arguing. It is interesting to notice that a preliminary debate at the court was on the rationality of the decision process. The question was whether a judge or a jury would take the case. The proposal of a jury was rejected because it was considered more emotional whereas a judge was considered more rational.

[3] J-Ph. Bouchaud. Economics needs a scientific revolution. *Nature*, 455:1181, oct 2008.
[4] John Vidal, McLibel, Burger Culture on Trial, *Macmillan Publishers*, 1997

3 Equilibria in games

In all escalation processes, there is an interaction–competition mechanism and competition prevails. Early philosophers noticed that the rules that govern group activities are those of games, leading to the development of *game theory*. Indeed like in an actual game, players cooperate more or less, but overall they act for their own interest. Early on, the concept of game describes how the interaction between the actors of a system works. This applies particularly to economics. The founding act of game theory is John von Neumann and Oskar Morgenstern book, *Theory of Games and Economic Behavior*, published in 1944.[5]

Jean-Jacques Rousseau tells in his *Discourse on the Origin and Basis of Inequality Among Men* (1775) how even acculturated men act collectively mixing interaction and selfishness. This leads them to choose an option fitting with their best immediate interest. He chose the example of a deer hunt which remains infamous.

> "In this manner, men may have insensibly acquired some gross ideas of mutual undertakings, and of the advantages of fulfilling them: that is, just so far as their present and apparent interest was concerned: for they were perfect strangers to foresight, and were so far from troubling themselves about the distant future, that they hardly thought of the morrow. If a deer was to be taken, every one saw that, in order to succeed, he must abide faithfully by his post: but if a hare happened to come within the reach of any one of them, it is not to be doubted that he pursued it without scruple, and, having seized his prey, cared very little, if by so doing he caused his companions to miss theirs."

In this duality collective – individual involvement, the agent adopts a course of action from which she has no interest to deviate. A strategic position, where individual behaviors are stuck because agents do not change their choices is called an *equilibrium*. In his book *Researches on the Mathematical Principles of the Theory of Wealth* (1838), the economist and mathematician Antoine-Augustin Cournot evidenced in the case of duopoly this notion of equilibrium which has not yet its name. Two companies compete for production of the same objects and try to adjust their production to optimize their profits. A Cournot equilibrium is the optimum quantity which companies must manufacture to earn the most money. One of the characteristics of Cournot duopolies and Rousseau hunters is the interaction without cooperation. We will therefore focus on non cooperative games, in which each player is selfish and does not attempt to help the others even thought this may help her in the long term. Keeping the framework of non cooperative games, we go from Antoine A. Cournot to John F. Nash, who in 1947 stated a general form of equilibrium which one calls *Nash equilibrium* and which started an active research on non cooperative games. These games are defined on a particular form of games which we call *normal form games*. They are one shot game where payoff are immediately distributed. A typical example of normal form game is the game *stone-paper-scissors*. Two players show at the same time one of the three following objects, a stone, a paper or scissors. The rules are as follows:

- paper beats stone,
- stone beats scissors,

[5]The first paper on game theory is usually attributed to Ernst Zermelo and entitled *"Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels"*, in the Proceedings of the Fifth International Congress of Mathematicians (1913).

- scissors beat papers.

In this game each player must randomize her choices and her optimum strategy is to play 1/3 paper, 1/3 stone and 1/3 scissors. The game *matching pennies* is a simplified version of the game *stone-paper-scissors*. Two players whom we call Alice and Bob display at the same time a coin, *head* or *tail*.

- If both players display *heads* or both players display *tails*, Alice wins.
- If both players display mismatching pennies, Bob wins.

Again players must play head and tail with equal probability. We will not extend on this type of games, because there is too much emphasis given to the value of the payoff, which is at the core of probability and we claim that the actors play without quantifying their gains. We are interested by sequential games with no probability, as proposed by Harold Kuhn in 1953, following Zermelo. In what follows values attributed to players will be symbolic. No computations are required, only comparisons.

4 Sequential games

In a sequential game, players play several runs, each player at her turn. The distribution of the payoffs takes place at the end of the game. In these games, there are many kinds of equilibria, but those we are interested in are the so-called *backward equilibria*, since they translate the rationality of the choices of the players. They are called "backward" since they compute backward from the end of the game. Without loss of generality, we consider only two player games. For instance, consider a variant of the game *matching pennies*, where this time, players Alice and Bob play one after the other. More precisely, Alice plays first, then Bob, then Alice again. In case of a matching, two heads in a row or two tails in a row, Alice wins the set. In the other case Alice looses her sets. The scores are accumulated and counted at the end of the game. We count the global win, loss or tie which corresponds respectively as w, ℓ or t. For instance, if Alice plays head, then Bob plays tail, and Alice plays head, there is no matching, then Alice globally looses and Bob globally wins, resulting in ℓ. If Alice plays head, Bob plays tail and Alice plays tail, there is a global tie for both resulting in t.

This games has an almost obvious winning strategy, but we consider it only as an illustration, in order to show that we must consider even the most stupid strategies, according to the counterfactuality.[6] Very naturally this game is presented by the diagram of Figure 1 where Alice is identified by A and Bob is identified by B and where arrows are the steps of the game. Label h of an arrow shows that the player plays *head*, whereas label t shows that player plays *tail*. Remind that the value of the payoffs has no meaning but the fact that they can be compared. Thus $0 < 1$ and $1 < 2$. Number 0 means that player never won, number 1 means that player won as often as her opponent and number 2 means that player won twice more that her opponent.

On the diagram, each player has two choices: either to go down (head) or to go right (tail). Starting at the leftmost uppermost node, following the arrow labeled h, then the arrow labeled t, then the arrow labeled h, one gets to the end of the game. The tag ℓ means "Alice globally looses and Bob globally wins". Let us call $h\,t\,h$ such a strategy profile.

[6] A reasoning is counterfactual if it relies on hypotheses which are not necessarily plausible. "If there are Martians, they try to communicate with us".

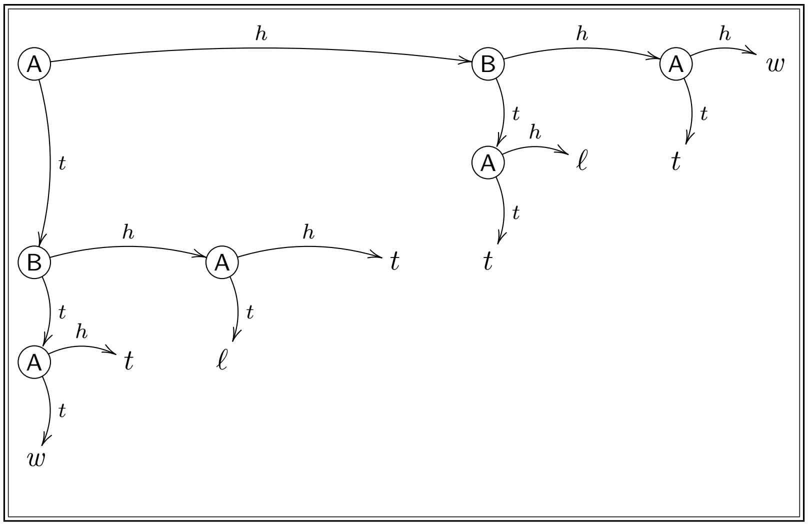


Figure 1: The matching pennies sequential game, where A starts, then B plays, then A plays again.

In this game, among the eight strategy profiles, two are equilibria (Figure 2). Thus in the strategy profile $h\,t\,t$ which leads to t if Bob changes his choice in h, then Alice would chance her choice in h and he would loose, therefore he would not do it. The computation of such an equilibrium requires what specialists call a *backward induction*. Let us consider a position in the game and let us look at positions which come after (following the arrows). If we consider this configuration, we see that it is itself a game (a *subgame*) completely included in the game of Figure 1. The idea is to attribute to each subgame a couple of payoffs which corresponds to what the equilibrium returns. To associate these couples one starts from the end of the game and one goes back to the start of the game. Step by step, one builds the assignments of the payoffs for the subgames. We start from the ends of the game, then we build slightly larger games, then still larger, to get to the largest game, that is the game we are interested in. In other words, we build equilibria gradually. At each step, we choose for the player of interest, the larger payoff and we remember the choice. The equilibrium is the strategy profile which goes through the taken positions.

Characteristics of the analysis of sequential games is *counterfactuality*. We reason in all situations as they would be possible. Clearly, some game positions, located after one player has chosen another option will not take place, but they must be taken into account to explain how the players reason. Counterfactuality plays an even more important role in infinite games so crucial in escalation.

Which link between backward equilibria and rationality? Robert Aumann, Nobel Prize winner in economics in 2005, has shown in 1995 that backward equilibria are rational strategy profiles. They are strategy profiles in which everybody knows that nobody can change her choices without loosing or drawing and everybody knows that rationally no other case can appear. We say that the players choose these equilibria from a common knowledge of the rationality of the other players.

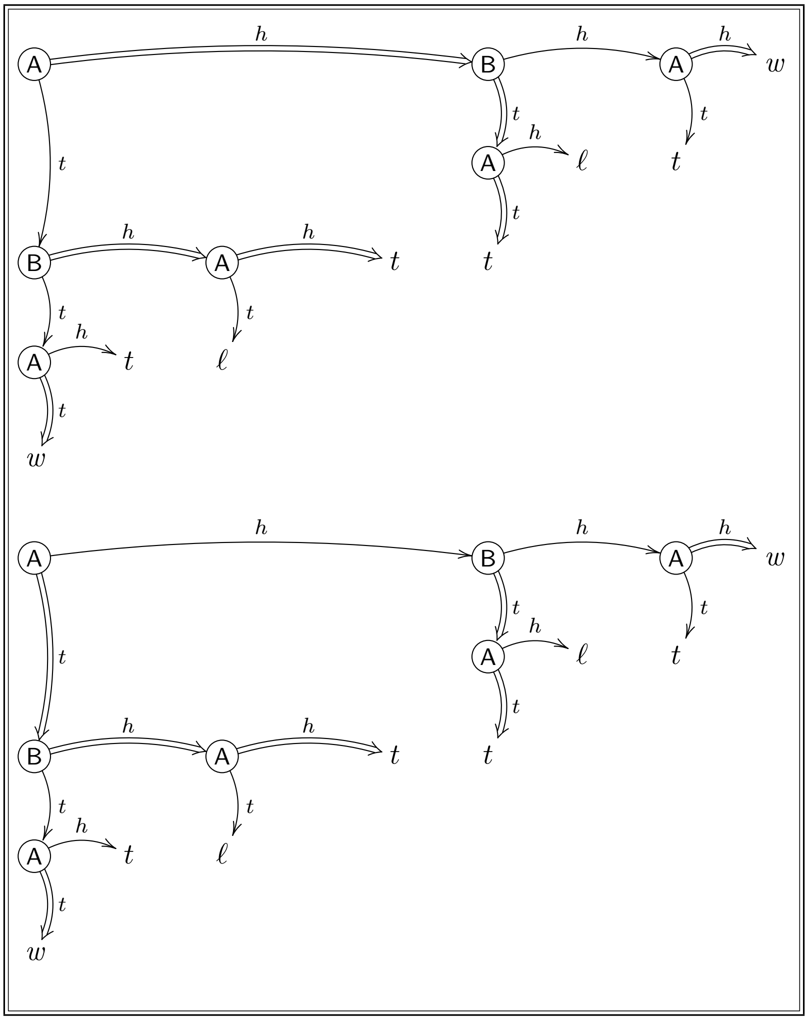


Figure 2: The two equilibria of the matching pennies. *The double arrows correspond to choices made by players*

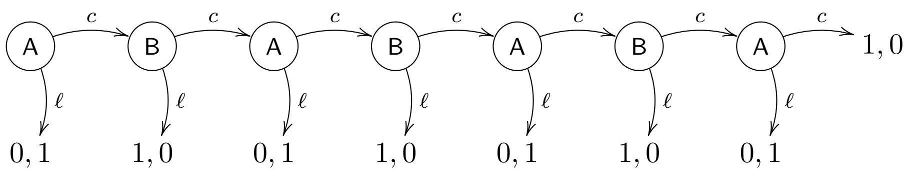
5 0, 1 Games

Vulnerant omnes

Ultima necat

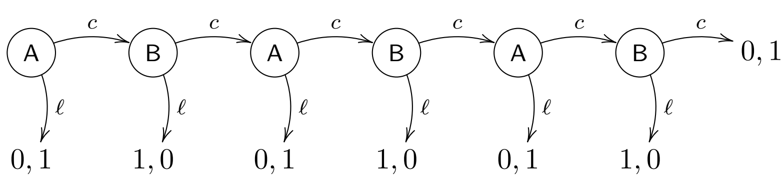
Sentence disposed on sundials,

We will focus on simple sequential games. In these games, players have choices to leave (ℓ) or to continue (c). The payoffs are 0 € ou 1 €[7] Let us focus on the game with seven players.



- At the last turn, Alice must of course choose c, since she earns 1 €.
- At the penultimate turn, Bob may choose, what a choice! Indeed, if he continues he earns nothing and if he leaves he earns nothing as well.
- At the fifth turn, Alice continues because in each case, she earns 1 €. And if she leaves she earns nothing at all.
- At the fourth turn, Bob has the same choice as at the penultimate turn, namely to earn nothing or to earn nothing.
- At the third turn, Alice continues.
- At the second turn, Bob must choose between nothing or nothing.
- At the beginning of the game, Alice should do nothing by stop

The $0, 1$ game has seven turns and more than one rational strategy profile. They all have the characteristics that Alice always continues and Bob does whatever he wants. Let us consider now the $0, 1$ game with six turns.



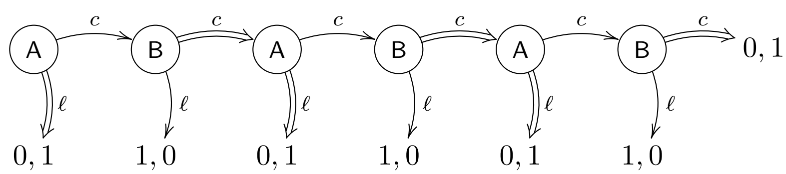
This time the backward induction yields something different.

- At the last turn, Bob continues.

[7]To spice up the game, we could replace 1 € by 1 000 000 000 € and therefore say that the players either receive nothing or receives a billion of euros. We insist on the fact that the quantity does not count, only counts the comparison.

- At the penultimate and fifth turn, Alice does whatever she wants.
- At the fourth turn, Bob continues.
- At the third turn, Alice does whatever she wants.
- Etc.

Here is the diagram of the game where Alice leaves always.



In this game, and in all $0, 1$ games with an even number of turns, the rational strategy profiles, that are the equilibria, are when Bob continues always and Alice does whatever she wants. We will look at infinite $0, 1$ games and see what the equilibria in those games are.

6 The dollar auction

both
Walk toward each other and the duel starts again
Then by degrees in its dark dementia
The battle intoxicates them; then come to their heart back
This I don't know which god who wants that one is a winner.

Victor Hugo,
La Légende des siècles. Le mariage de Roland

Honi soit qui mal y pense

King Edward III,
Motto of the Order of the Garter

The dollar auction game has been described by Martin Shubik[8] in 1971 and it is well known in France under the name *American auction*. There it is made in some weddings and it consists in selling the garter of the bride, the products of the sale helps the couple for their honeymoon or for their settlement. In his version Shubik sales through an auction one dollar and the goal is to collect much more than the price of the object. The dollar auction consists in selling the object (the dollar) as follows. Each time a person bids for n €, she must put the given amount in a hat and this amount is never returned to her. As Shubik wrote it is suggested to wait for starting the auction that "the spirits are high", as it is the case at the end of a wedding party and we can limit ourself to two bidders, because this does not change the phenomenon. Actually in most of the cases,

[8] M. Shubik, The dollar auction game: A paradox in noncooperative behavior and escalation. *Journal of Conflict Resolution*, 15(1):109–111, 1971.

even though it starts with more than two bidders and an increment of 1 €, the auction ends with two bidders who do not want to give up. Then one notices an escalation phenomenon. The bidders are going to pay more than the value of the object and more than what they wanted to invest at the beginning. They keep fighting in order to acquire the garter or the George Washington bill. Actually they have invested so much that they do not want to give up without getting the desired object.

In Shubik analysis and in this of his successor, we see two contradictory statements:

- *To have escalation, the game must be infinite.* More precisely, Shubik writes that "the analysis is confined to a (possibly infinite) game without a termination point, as no particular phenomenon occurs if an upper bound is introduced".
- *Shubik and his followers analyze finite games*, then extrapolate their results to infinite games.

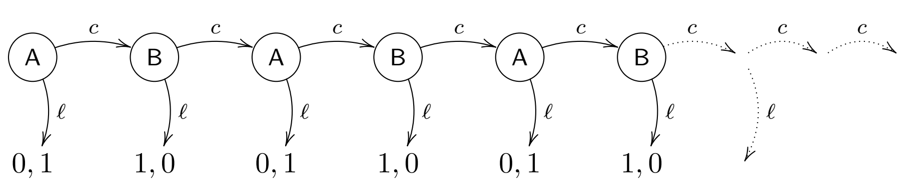
From that, they conclude that the only equilibrium is this where no player starts the auction and never bids. They claim that the rational player would reject at any price any type of escalation. We have shown that this strategy profile is not an equilibrium in the dollar auction. To extrapolate finite games to infinite games, as Shubik and those who have adopted his reasoning do, is wrong. We will come back on it later.

7 The infinite sequential games : back to the $0,1$ game

He walks on the immense plain
Goes ahead, comes back, throws the grain further up
Reopens his hand, and starts again,
And I meditate, obscur witness.

Victor Hugo,
Saison des semailles. Le soir

In $0,1$ games we have limited the number of turns. But nothing prohibits to consider infinitely many turns.



What are the equilibria in this case? One can no more define the equilibria by backward induction, but we can use a similar method[9] which assumes that we reason on infinite objects. Let us call it *backward coinduction*. Actually there are at least two equilibria:

1. Alice leaves always and Bob continues always (Figure 3),
2. Alice continues always and Bob leaves always (Figure 4).

[9] The specialists following Selten (1965) speak of *subgame perfect equilibria*, but we propose to call them *backward coinduction equilibria*.

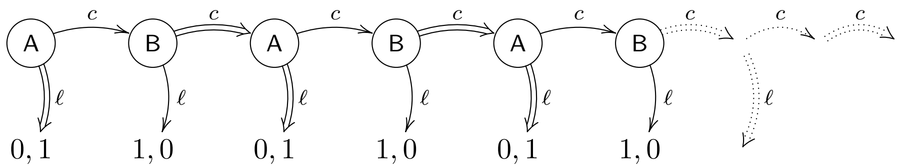


Figure 3: Alice leaves always and Bob continues always.

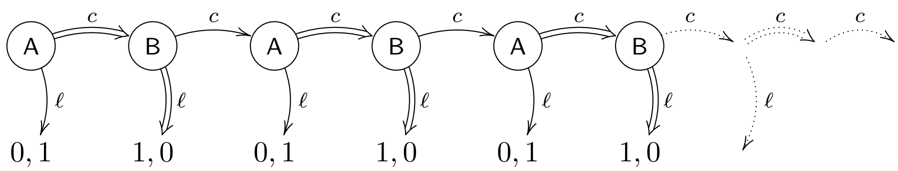


Figure 4: Alice continues always and Bob continues always.

Justification *This paragraph can be skipped at first reading.* Let us see why and how this works. Let us recall how we proceeded in the case of finite games. Starting of end games, we examine a game position. There are a player and two subgames of which we know at least an equilibrium: we choose as an equilibrium for the whole game the one that corresponds to the player making the choice of the best of the two equilibria. Now let us go to infiniteness and let us show that the strategy profile where Alice always leaves and Bob always continues is an equilibrium. Like in the case of the backward induction, we know the equilibria for the subgames. In the case where it is Bob's turn, what are the equilibria? There are two, one is Bob's abandon and in the other, where Bob continues, one has a sub-equilibrium in which Alice starts. What is the equilibrium where Alice starts? This is the same as this we are looking for, namely the strategy profile where Alice always leaves and Bob always continues. Among the optimal strategy profiles available to Bob, which one is the best? This where Bob continues, since Alice abandons and he wins 1 €, wherever if he would leave, he would earn nothing. Hence the good choice, which yields the equilibrium, is this where Bob continues, then Alice leaves always, then Bob continues always. In the case of Alice's turn, a similar reasoning shows that an equilibrium is this where Alice always leaves and Bob always continues. Let us summarize: We have shown that if one makes the hypothesis that the equilibrium is the strategy profile where Alice always leaves and Bob always continues then the equilibrium is the strategy profile where Alice always leaves and Bob always continues. This a bit sophisticated explanation may be better understood on Figure 5 where we have represented the game $0, 1$ more compactly. Are we cycling? No! Our reasoning is perfectly correct, because the hypothesis is on strict subgames. Let us call it *backward coinduction*: induction comes from the noun *induction* and the prefix *co*, "associated", and *backward* insists on the analogy with backward induction. Actually we have shown that the strategy profile where Alice leaves always and Bob continues always is an equilibrium all the game along. We say that this is an "invariant" of the infinite game.

The same kind of reasoning applies to the dollar auction. This is slightly more complex, since the invariant is not the same strategy profile, but the same parametrized strategy profile to take into account the fact that the involved numbers increase. Then there are at least two equilibria in the dollar auction: one for Alice continuing always and Bob leaving always and the other for the other way around.

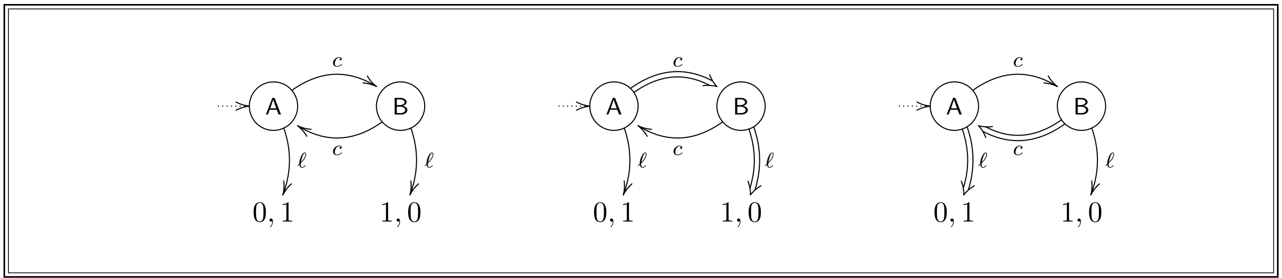


Figure 5: The $0, 1$ game and its equilibria seen compactly

8 Escalation is rational

Irrational exuberance has unduly escalated asset values.

Alan Greenspan,

Federal Reserve Board Chairman, on May 12 1996

We know equilibria of the $0, 1$ game, but how this may lead to escalation? Assume that our players or economic actors are rational, but have no memory and no ability to reconsider their choice. They forget immediately and learn nothing from past. They know how to analyze a situation and understand their interest, but they do not know how to take advantage from their experience, they do not see themselves reasoning and they have no reflection on their own reasoning. Moreover they do not take into account the marginal fees and they do not see a moving society. In short they are introvert and forgetful. At each step of the $0, 1$ game, Alice starts a new infinite $0, 1$-game. She knows she has two possible rational strategies. The first strategy consists in continuing today, tomorrow, the day after tomorrow and always, assuming that Bob will leave always. In this second strategy, she takes very seriously the resolution of Bob to continue always. This is sometime called a *credible threat*, which is the attitude of Morris and Steel's friends[10] in front of MacDonald's. Alice will continue if she thinks that she impressed enough Bob to scare him and make him to abandon always. This is MacDonald's attitude all along the case against Morris and Steel. "This is not possible. They are going to give up" thought MacDonald's board. If we are in a situation where no one takes seriously the threat of the other, no threat is credible, we are in an escalation.

Let us recall two characteristics of escalation. Like in any sequential game, only comparisons[11] are pertinent. Agents handle entities that are completely abstract: they do not know the value of what they handle. They know only how entities are compared. Moreover, agents are faced at each step to two (or more) options equally rational, therefore the evolution of the process is highly unpredictable. At this level exogenous influences on the decision process may happen. Since Alice has no objective reason to choose between "continuing" or "leaving", she can take into consideration emotional aspects[12] or invoke other rational criteria.

[10] Among five pamphlet distributors, only Morris and Steel fought back MacDonald's.

[11] In any case, using probabilities, assuming we know on what it applies, would not add anything of the prescription in successive choices.

[12] Daniel Kahneman, *Thinking, Fast and Slow*, Farrar, Straus and Giroux, 2011.

9 Escalation and cognitive psychology

It is worth wondering whether agents are really rational. Take for this the view of cognitive psychology as describe in Keith E. Stanovich *What Intelligence Tests Miss: the psychology of rational thought*.[13] A rational agent owns a *mindware* made of rules, knowledge and procedures which he can retrieve in her memory and which she acquired by a mental training and/or by education and which allows her to make decision and to solve problem. Of course we assume that the mindware of the agents we consider contents coinductive reasoning tools or all types of equivalent reasonings which allows conducting correct deductions on infinite mathematical objects.[14]

There are two kinds of rationality from the coarser one to the finer one. On one side the instrumental rationality allows the agent to behave appropriately among the world so that she gets what she wishes, using her physical and mental resources. The economists and the cognitive scientists refined the notion of wished goal to this of *expected utility*. The *epistemic rationality* lies above instrumental rationality and interacts with it. It allows the agent to confront her set of beliefs to the effective structure of the world. We would say that it makes the agent able to think about her own way of reasoning. More conventionally, we would say that the epistemic rationality deals with what is true, whereas instrumental rationality deals with actions to maximize aims. The first form of rationality corresponds to algorithmic mind and the second one to reflexive mind. A reflexive mind is able to analyze how she reasons.

We claim that an escalating agent owns a correct algorithmic mind, but meanwhile she lacks a reflexive mind which would allow her, by revising her belief, specifically her belief in an infinite resource, to escape from the escalation spiral. Indeed at the beginning such a belief gives her dynamism for investing, but as the adage says “Errare humanum est, perseverare diabolicum”. Therefore there is a time when a rational agent must understand that a belief in an infinite resource leads her to a dead end and that her judgement must be revised. Doing so, early enough, the agent shows that she is really rational.

10 The ubiquity of escalation

> She bought ostrich feathers, Chinese porcelain, and trunks; she borrowed from Félicité, from Madame Lefrançois, from the landlady at the Croix-Rouge, from everybody, no matter where. With the money she at last received from Barneville she paid two bills; the other fifteen hundred francs fell due. She renewed the bills, and thus it was continually.
>
> *Gustave Flaubert*
> Madame Bovary

Escalation is a very frequent phenomena as soon as participants are rational and consider infinite resources. In some cases this is what people look for, because this is a survival condition, like in the

[13] Yale University Press 2010. One can also read Antonio Damasio *Descartes' Error: Emotion, Reason, and the Human Brain*, Putnam, 1994

[14] Stanovich seems to not be aware of coinduction and therefore considers escalation as dysrational in his mindware. Nevertheless his distinction between instrumental and epistemic rationality remains operational.

case of evolutive biology. In his *Red Queen* theory Leigh Van Valen describes the competition of two species and the survival condition of each species that results, namely a continual adaptation to fight the challenge of the species. Escalation is not a drawback, but a positive quality that makes the species perennial when it has a challenger. In another hand, in economy, we know perfectly well the consequence of escalation, namely speculative bubbles with huge devastations when they blow up.

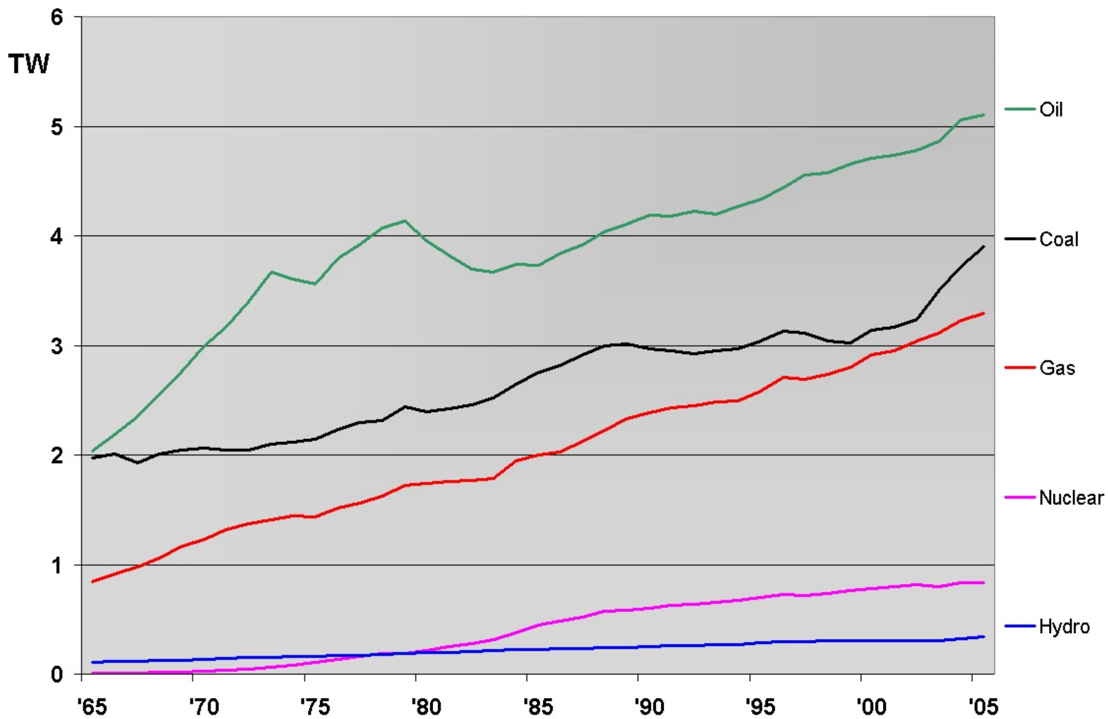


Figure 6: Evolution of world energy consumption (source Wikipedia)

Development theory says that earth provides us with a finite amount of fossil energies and the sun light is a limited resource as well. Therefore the increasing of energy consumption (Figure 6) is a frightening escalation.

In another field, escalation is a consubstantial component of war. From the Bear Hall Putsch to his suicide, through *Mein Kampf* publication and Stalingrad battle, Hitler's trajectory is an escalation. Locally and in isolation, Hitler has displayed a strategic rationality, i.e., an instrumental rationality.

11 It is not possible to extrapolate

One morning we set sail, with brains on fire,
And hearts swelled up with rancorous emotion,
Balancing, to the rhythm of its lyre,
Our infinite upon the finite ocean.

⋮

We want, this fire so burns our brain tissue,
To drown in the abyss – heaven or hell, who cares?
Through the unknown, we'll find the new.

Charles Baudelaire
Le voyage *(Travel)*

In Section 7, we saw that the reasoning mistake of Shubik and mostly of his followers was to prune away an infinite branch of an infinite game to make finite games, then to reason on finite games, then to extrapolate the results obtained on the finite games to the infinite game. We have seen that on the $0, 1$ game this method cannot work: according to the fact one prunes away at an even step or at an odd step the results are different and there is no natural way to extrapolate. In the case of the dollar auction, the problem is more vicious since the results look consistent on the different size of infinite games. However it may depend on the way the cut is done. What is wrong in the extrapolation has been well identified by the mathematician Weierstrass in 1871: what we know on the finite does not foretell what we can say on the infinite. More precisely, he has proved that a well known and classical property of finite sums of functions (to be differentiable everywhere and to be associated with smooth curves), disappears for infinite sums of functions (which can be nowhere differentiable and be associated with especially rough curves). This result was a surprise when it has been published, since great mathematicians before him admitted without proof the persistence at infinity of the differentiability. Moreover, this evidenced the existence of monster curves without tangents, which have been systematically studied by Mandelbrot as fractals (Figure 11). Actually this mistake on extrapolation goes back to Zeno of Elea, who stated that Achilles would never overtake the tortoise. This led to negate motion. Indeed Zeno extrapolated the true result on one run that Achille does not overtake the tortoise, because he started after the tortoise and reached the point where the tortoise started. There are infinitely many such runs. Zeno is wrong when he extrapolates his result, true on one run, to the limit of the infinite sequence of runs. He should not conclude that Achille will not overtake the tortoise.

Let us consider another example, namely a recent and easy to understand mathematical result, founded on concepts known from Pythagoras and Euclid. An *arithmetic progression* is a sequence obtained from some initial term by adding always the same *common difference*. The sequence $5, 8, 11, 14, 17, ...$ is an arithmetic progression of common difference 3. There exists no infinite arithmetic progression made only of primer numbers.[15] Ben Green and Terence Tao have proved

[15] A prime number is a number divisible by the two numbers 1 and itself. If the origin of the progression is n and its common difference is d, then the $(n+1)^{th}$ element is $n + n \times d$ is clearly not prime, because it is divisible by n.

in 2004 that there exist arbitrary long finite arithmetic progressions made only of prime numbers. This difficult result deserved Tao the Fields medal. We face clearly a result that is not extrapolable. The very specific interest of this result is that the finite case is incommensurably more difficult that the infinite case. We were more used to the opposite case, namely when the finite case is easier than the infinite case which requires the subtle coinduction.

12 Conclusion

By a precise analysis of the infinite, we have shown that agents involved in an escalation are (instrumentally) rational. Consequently, any approach that affirms too fast that they are irrational[16] has missed the right argument based on a coinductive analysis of infinite games. The use of coinduction and more generally of reasonings on infinite objects should be part of the new foundation of economics or as cognitive psychology scientists would say, coinduction must be part of the mindware of the economists. In this framework, equilibrium will not be paired with stability.

On another hand the internal rationality of the agent should be made distinct from the external rationality of the observer. The agent who stipulates an infinite availability of resource is introvert. She sees only her own short term interest and lacks of "reflection". She is not able or she is not willing to imagine a global analysis, whereas the observer sees immediately the behavioral aberration of the system. Each one has his own rationality and the points of view cannot be reconciled.

[16]We do not deny they are party irrational. Indeed bewildered, they can invoke irrational arguments to raise perplexity.

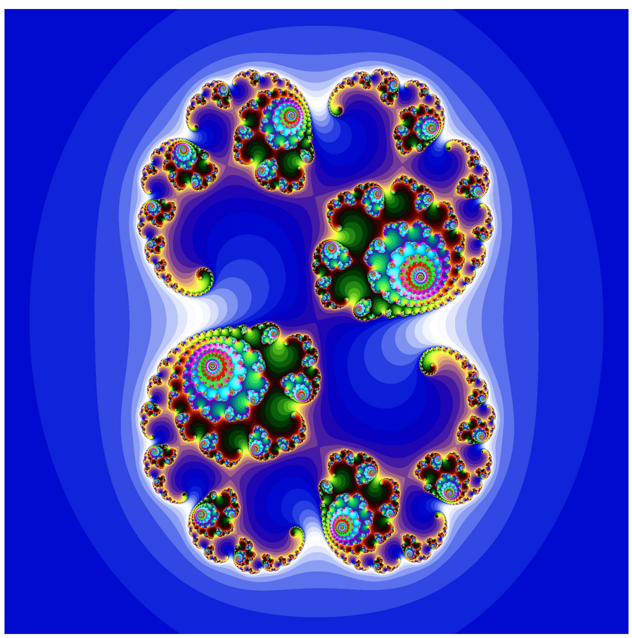


Figure 7: The Julia set, with a bound which is a tangentless curve

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This paper presents the mathematical and logic substratum of what is presented here together with a detailed and complete bibliography.

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This paper presents for specialists of coinduction the problem of escalation and of its rationality.